D 12425 (Pages : 4)

Name..............................

Reg. No..............................

Original

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(CUCBCSS–UG)

Complementary Course

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*

Each question carries 1 mark.

1. Verify that $y = cx^3$ is a solution of $xy' = 3y$.

2. Solve $y' = -x/y$.

3. Test for exactness $2xydx + x^2dy = 0$.

4. Define rank of a matrix.

5. Find the characteristic roots of the matrix $A = \begin{bmatrix} -7 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.

6. State the Cayley-Hamilton Theorem.

7. If $\vec{p}, [3, \; 2, \; 0]$ and $[-2, \; 4, \; 0]$ are in equilibrium, find $\vec{p}$.

8. Find the gradient of the scalar function $f(x, y, z) = xyz$.

9. Illustrate commutativity of vector addition with an example.

10. Define a simply connected domain.

11. State Green's theorem in the plane.

12. Give a parametric representation of the sphere.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $\left(x^2+y\right)dx+\left(y^2+x\right)dy=0$.

14. Solve $y'-y=4$.

15. What is the characteristic equation of $\begin{bmatrix}1 & 2\\ 7 & 1\end{bmatrix}$?

16. Solve $dy/dx+y/x=x^3$.

17. Using Cayley-Hamilton theorem, find A^2 if $A=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$.

18. Find the rank of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}$.

19. Find a parametric representation of the straight line through (4, 2, 0) in the direction of [1, 1, 0].

20. Find a tangent vector and unit tangent vector for $\vec{r}(t)=\left[t, t^3, 0\right]$.

21. Show that the form under the integral sign is exact :

$$\int e^{x-y+z^2}\left(dx-dy+2z\,dz\right).$$

22. Give the standard form of the Bernoulli equation with a suitable example.

23. Using Green's theorem, find the area enclosed by the ellipse :

$$x^2/a^2+y^2/b^2=1.$$

24. Find curl of $\left[y/z, x/z, -xy/z^2\right]$.

(9 × 2 = 18 marks)

Section C

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find an integrating factor and solve :

$$\left(x^2-2x+2y^2\right)dx+2xy\,dy=0.$$

26. Find the Orthogonal Trajectories of $y=ce^{-x}$.

27. Reduce to normal form and find the rank of $A=\begin{bmatrix}1&1&1&1\\1&2&3&4\\4&3&2&1\end{bmatrix}$.

28. Solve : $x+2y+3z=0$.
$2x+y+3z=0$
$3x+2y+z=0.$

29. Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$.

30. Find the length of the vector $\vec{r}(t)=[t,\cosh t,0]$ from $t = 0$ to $t = 1$.

31. (i) Prove that $\operatorname{curl}(\operatorname{grad} f)=\vec{0}$ for any twice differentiable scalar function 'f'.

(ii) Prove that $\operatorname{div}(\operatorname{curl}\vec{v})=0$ for any vector $\vec{v}$.

32. Use Gauss Divergence Theorem to evaluate :

$$\iint_S \vec{F}\circ\vec{n}\,dA;\ \vec{F}=\left[x^2,0,z^2\right]\ \text{S is the box } |x|\le 1, |y|\le 3, |z|\le 2.$$

33. Find the work done by a Force $\vec{F}=\left[y^2,-x^2,0\right]$ in moving an object along the straight line segment from (0, 0) to (1, 4).

(6 × 5 = 30 marks)

Turn over

Section D

*Answer any **two** questions.*
Each question carries 10 marks.

34. Find the characteristic roots and any two characteristic vectors of :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

35. (i) Find the tangential and normal acceleration of $\vec{r}(t) = [0, 0, 5t^2]$.

(ii) Find the directional derivative of $f(x, y) = x^2 + y^2$ at (1, 1) in the direction of [2, – 4].

(iii) Given $\vec{v} = \text{grad } f$, find 'f' if $\vec{v} = \left[\frac{y}{z}, \frac{x}{z}, \frac{-xy}{z^2}\right]$.

36. State Stokes Theorem. Verify it for :

$\vec{F} = [z^2, 5x, 0]$, S is the square $0 \le x \le 1, 0 \le y \le 1, z = 1$.

($2 \times 10 = 20$ ma

D 71693 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write the general form of first order ODE.
2. What do you mean by exact differential equation ?
3. Define dot product of two vectors.
4. State Cayley Hamilton theorem.
5. When will you say two matrices are equivalent ?
6. Define curl of a function.
7. Find the resultant of the vectors $p = [2, 4, -5], q = [1, -6, 9]$.
8. Define characteristic polynomial of a matrix.
9. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^3 + 8x = 0$.
10. Find the directional derivative of $f = x^2 + y^2$ at (1, 1) in the direction of $2i - 4j$.
11. Write the general form of Bernoulli differential equation.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Verify that $\frac{c}{x}$ is a solution of the differential equation $xy' = -y, c$ is a constant and $x \neq 0$.
14. Find the curve through the point (1, 1) in the xy-plane having at each of its points the slope $-\frac{y}{x}$.

Turn over

15. Solve $2xyy' = y^2 - x^2$.

16. Let $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find the distance between u and v.

17. Find the projection of $a = [1, -3, 4]$ in the direction of $b = [3, 4, 7]$.

18. Find the unit tangent vector T to the curve $C = F(t) = (t^2, 3t-2, t^3, t^2+5)$ in R^4 when $t = 2$.

19. Find the component of (1, 1, 3) in the direction of (0, 0, 5).

20. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x) = 2x^3 - 4x + 5$. Find $f(A)$.

21. Show that $\text{curl}(u+v) = \text{curl } u + \text{curl } v$.

22. Show that every elementary matrix E is invertible, and its inverse is an elementary matrix.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} .e^x (\cos y dx - \sin y dy)$ is path independent.

24. Find the length of the curve $r(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find all the curves in xy-plane whose tangents pass through the point (a, b).

26. Solve $\cos(x+y)dx + (3y^2 + 2y + \cos(x+y))dy = 0$.

27. Find an integrating factor and solve the initial value problem
$(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, y(0) = -1$.

28. Find the straight line L_1 through the point P : (1, 3) in the xy-plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

29. Find the volume of the tetrahedron with vertices (0, 0, 0), (1, 2, 0), (3, –3, 0), (1, 1, 5).

30. Show that the integral $\int_c F.dr = \int_c 2xdx + 2ydy + 4zdz$ is path independent in any domain in space and find its value in the integration from A : (0, 0, 0) to B : (2, 2, 2).

31. Describe the region and evaluate $\int_0^1 \int_{x^2}^{x} (1 - 2xy)\, dydx$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1+\cos\theta)$.

33. Verify Greens theorem in the plane for $F = \left[-y^3, x^3\right]$ and the region is the circle $x^2 + y^2 = 25$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Let $A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}$.

(a) Find all eigen values of A.

(b) Find a maximal set S of non-zero orthogonal eigenvectors of A.

(c) Find an orthogonal matrix P such that $D = P^{-1} AP$ is diagonal.

35. Solve :

(a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$.

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate the integral by divergence theorem $F = \left[z - y, y^3, 2z^3\right]$, S the surface of $y^2 + z^2 \le 4, -3 \le x \le 3$.

(2 × 10 = 20 marks)